

Product Rule proof (& useful technique) <sup>day 24</sup>

$$\lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - \cancel{f(x)g(x+h)} + \cancel{f(x)g(x+h)} - f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x+h)}{h} + \lim_{h \rightarrow 0} \frac{f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} g(x+h) \left[ \frac{f(x+h) - f(x)}{h} \right] + \lim_{h \rightarrow 0} f(x) \left[ \frac{g(x+h) - g(x)}{h} \right]$$

$$= g(x)f'(x) + f(x)g'(x)$$

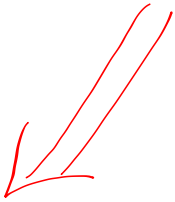
$$\left| \text{so } \frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \right|$$

day 24

Play

$$\frac{d}{dx}(x^2 \cdot x) = \frac{d}{dx}(x^2) \cdot x + x^2 \frac{d}{dx}(x)$$

$$= 2x^2 + x^2 = 3x^2$$


$$\frac{d}{dx}(x^3) = 3x^2$$

day 24

3.3/39) Let  $f(x) = x^2 - 6x + 5$

a) Find ... slope is 0

Find the values of  $x$  for which the slope  
of the curve  $y = f(x)$  is 0

direct object

$$f'(x) = 2x - 6 = 2(x - 3)$$

$$f'(x) = 0 \Rightarrow x = +3$$

done :)

33

day 24

49/ a)  $\frac{d}{dx}(10^5) = 5 \cdot 10^4$  FALSE  $\frac{d}{dx}(1) = 0$   
 $5 \cdot 10^4$  however, this is not the same as  $50^4$

b) the slope of a line tangent to  $f(x) = e^x$   
 is Never 0.  
 true, because  $\frac{d}{dx}(e^x) = e^x$ , and that is never 0

c)  $\frac{d}{dx}(e^3) = e^3$  false, because  $e^3$  is just a number  
 $\frac{d}{dx}(e^{3x}) = \frac{d}{dx}(e^x \cdot e^x \cdot e^x) =$  use product rule

d)  $\frac{d}{dx}(e^x) = x e^{x-1}$  FALSE. don't be an idiot,

e)  $\frac{d^n}{dx^n}(5x^3 + 2x + 5) = 0$ , for any  $n \geq 3$  false  
 (I could make it  $n \geq 4$ ...)

$$\frac{d}{dx}(5x^3 + 2x + 5) = 15x^2 + 2$$

$$\frac{d^2}{dx^2}(5x^3 + 2x + 5) = \frac{d}{dx}(15x^2 + 2) = 30x$$

$$\frac{d^3}{dx^3}(5x^3 + 2x + 5) = \frac{d^2}{dx^2}(15x^2 + 2) = \frac{d}{dx}(30x) = 30$$

day 24

3.4/b show 2 ways to differentiate  
 $f(x) = (x-3)(x^2+4)$

$$i) (x-3)(x^2+4) = x^3 - 3x^2 + 4x - 12$$

$$\frac{d}{dx}(x^3 - 3x^2 + 4x - 12) = 3x^2 - 6x + 4$$

$$\begin{aligned} ii) \frac{d}{dx}[(x-3)(x^2+4)] &= \frac{d}{dx}(x-3)(x^2+4) + (x-3)\frac{d}{dx}(x^2+4) \\ &= (1)(x^2+4) + (x-3)(2x) \\ &= x^2+4 + (2x^2-6x) = 3x^2-6x+4 \end{aligned}$$

day 24

3.4/5)  $\frac{d}{dx}(e^{kx}) = ke^{kx}$   
 what values of  $k$ ?

3.4/4 2 ways to differentiate  $f(x) = \frac{1}{x^{10}}$

i)  $\frac{1}{x^{10}} = x^{-10}$ . Power Rule:  $\frac{d}{dx}(x^{-10}) = -10x^{-10-1}$   
 $= -10x^{-11}$

ii) Quotient Rule:  $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

so:  $\frac{d}{dx}\left(\frac{1}{x^{10}}\right) = \frac{\frac{d}{dx}(1) \cdot x^{10} - (1) \frac{d}{dx}(x^{10})}{(x^{10})^2}$

$= \frac{0 \cdot x^{10} - 10x^9}{x^{20}} = \left(\frac{-10}{x^{11}}\right)$